9. Conferences 1998

International Conference on Probability and Its Applications February 24–26, 1998 Taejon (Korea)

The venue of the conference is the Korea Advanced Institute of Science and Technology (KAIST) in Taejon. It is organized by the Center for Applied Mathematics of KAIST in conjunction with the Korean Mathematical Society, supported by the Korea Science and Engineering Foundation and the Korea Research Foundation. The conference will be held under the auspices of the EAPRC.

Programme Committee:
L.H.Y. Chen (Co-chairman), B.D. Choi (Co-chairman), T. Brown, M. Fukushima, T. Hida, C.R. Hwang, C.E.M. Pearce, J.A. Yan.

Local Organizing Committee:
K.S. Chang, B.D. Choi (Chairman), C.S. Choi (Secretary), D.M. Chung, J.W. Jeon, J.H. Kim, T.S. Kim, I.S. Wee.

Topics for the conference include: Limit Theorems for Stochastic Processes. Stochastic Analysis. Markov processes. Diffusions. Dirichlet Forms. Queueing Theory. Modelling of Telecommunication Systems. White Noise Analysis. Quantum Stochastic Analysis. Stochastic Models in Finance. Applied Probability Approximation. Long Range Dependence. Self-similar Processes. Statistics of Stochastic Processes. Stochastic Control. Random Sets and Fuzzy Processes.

The programme consists of invited lectures and twenty-minute contributed talks.

Invited speakers:
J.R. Artalejo, J.H. Dshalalow, T. Hida, T. Hsing, D.M. Chung, G. Kallianpur, N. Jain, Y.J. Lee, Y.M. Park, P. Pollett, S. Watanabe, J.A. Yan.

Abstracts for invited lectures and contributed talks should not contain more than 200 words each, and should contain author's affiliation and email address. They should be submitted (preferable by email) to the Organizing Secretary no later than December 1, 1997.

For the registration form, the abstract format and further information please check:

http://math.kaist.ac.kr/~cam

If you wish to attend the Conference, please complete the registration form and return it before January 1, 1998 to the Organizing Secretary:

C.S. Choi (Secretary)
Department of Mathematics
Korea Advanced Institute
of Science and Technology fax: +82-42-869-8194
Taejon 305-701, Korea phone: +82-42-869-2717
cam@queue.kaist.ac.kr, cam@math.kaist.ac.kr

Workshop in Mathematical Population Dynamics May 11-15, 1998 Gothenburg (Sweden)

The newly established Gothenburg Stochastic Centre is organizing a guest programme in Population Dynamics during April and May 1998. It will culminate in the announced workshop.

Topics emphasise on general properties of population growth, but special populations will also be discussed. Particular themes that have been mentioned include

- Dependence/interaction in population dynamics and branching processes.
- The process of extinction in a historical perspective: how gradual is “natural” extinction, as opposed to catastrophies?
- Quasi-stationarity and its role in the evolution of populations now extinct.
- Branching within branching and mitochondrial dynamics.
- Cell and molecular population dynamics more generally; the cell cycle.
- Meta-populations.

Participants will have a background in probability and statistics, in mathematical population dynamics and/or various branches of biology. The workshop is planned to consist of three days at the Gothenburg Mathematical Centre, and two at the Tjrn Marine Biological Laboratory, in the beautiful archipelago of the Swedish West Coast.

Programme committee:
Peter Jagers (Gothenburg), Mats Gyllenberg (Turku), Hans Metz (Leiden), Marek Kimmel (Houston), Ziad Taib (Gothenburg).

Local organizing committee:
Peter Jagers, Ziad Taib, Inga-Lill Sandman, Marina Alexandersson.

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Ziad Taib, Gothenburg

SPA'98
25th International Conference on Stochastic Processes and their Applications
July 5–11, 1998
Corvallis, Oregon (USA)

The World Wide Web page for the Conference which will be held on the campus of Oregon State University in Corvallis Oregon, has been updated:
http://www.orst.edu/dept/math/docs/spa98.html
Check it out! Visitors to this web site will find information about housing, registration, abstract submission and travel to Corvallis. Abstract submission, some lodging, and registration may be completed electronically. There is an APRIL 1, 1998 deadline, so don't delay!

Oregon and the Pacific Northwest offer diverse family vacation opportunities before and after the conference from mountains, rivers, lakes, oceans to sand dunes and the high plains desert. Links to vacation information are provided at the web site.
SO BOOKMARK THE SITE AND CHECK IT REGULARLY FOR NEW LINKS!
Our web site is the most complete source of information for SPA98. However, more limited information is available via email or by ftp to:

ftp.math.orst.edu

At login prompt type anonymous, use your full email address as password, and go to the directory /conferences/spa98 to get files.

For further information plaese contact

SPA'98
Department of Mathematics
Oregon State University fax: +1 503 737 0515
Corvallis, OR 97331
USA email: spa98@math.orst.edu

Prague Stochastics'98
6th Prague Symposium on Asymptotic Statistics &
13th Prague Conference on Information Theory, Statistical Decision Functions and Random Processes (joint session)
August 23–28, 1998
Prague (Czech Republic)

For the programme see the last issue of Bernoulli News or check the www site of the Conference.

For more information please contact:

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European Summer School
Markov Chain Monte Carlo Methods
August 7-10, 1998
Rebild (Denmark)

The scientific scope is to offer an advanced course on MCMC for bright and ambitious young postdocs and PhD students with backgrounds in statistics and probability.

Invited experts will introduce and exposit six topics within the area:

- Probabilistic aspects of MCMC
- Bayesian statistics and MCMC
- Complex models and MCMC
- BUGS/Graphical models
- New advances/special topics
- Perfect simulation.

The summer school web page can be found at:
http://www.maths.nott.ac.uk/hsss/Workshops/summerschool.html
The web page will be updated as a principal source of information.

For further information please contact:

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ICM'98
International Congress of Mathematicians
August 18–27, 1998
Berlin (Germany)

Responsibility for the scientific programme lies with the Programme Committee appointed by the International Mathematical Union. There will be about twenty one–hour Plenary Lectures covering recent developments in the major areas of mathematics and about 170 forty–five–minutes Invited Lectures in nineteen sections. There is one section on Probability and Statistics reflecting the importance of our field within mathematics nowadays. This was not always the case, at least not in the beginning years of the ICMs. On the next pages *Ulrich Krengel* from Göttingen provides a history of probability and statistics at the ICMs from the very beginning up to now.

For more information please check

http://elib.zib.de/ICM98

or write to

ICM'98
c/o Prof. Dr. J. Winkler
Technische Universität Berlin, MA 8-2
Fachbereich Mathematik fax: +49 30 314 21604
D-10623 Berlin, Germany email: icm98@zib.de

EMS'98
22nd European Meeting of Statisticians
August 12–18, 1998
Vilnius (Lithuania)

The meeting is also the 7th Vilnius Conference on Probability Theory and Mathematical Statistics, and will reflect the traditions of those meetings as well as those of the European Meetings of Statisticians. It will take place on the Vilnius University Campus, 11 Sauletekio av., Vilnius.

Organizing Committee:
Vytautas Statulevicius (Chairman), Volker Mammitzsch, Eugenijus Manstavicius, Henrikas Pragarauskas, Mifodijus Sapagovas, Leonas Saulis.
Secretariat: M.Bloznelis, A.Plikusas.

Programme Committee:
Peter Jagers, Sweden (Chairman); Peter Diggle, United Kingdom; Bronius Grigelionis, Lithuania; Ildar Ibragimov, Russia; Enno Mammen, Germany; Juni Palmgren, Sweden; Vygantas Paulauskas, Lithuania; and Elvezio Ronchetti, Switzerland.

The traditional special invited lectures at European Meetings of Statisticians will be given by

- Opening Lecture: Richard Gill, Utrecht
- Forum Lectures: Nancy Reid, Toronto
- Closing Lecture: Jean-Francois Le Gall, Paris
- Special Vilnius Lecture:
  Vidmantas Bentkus, Bielefeld.
- IMS Special Invited Paper:
  Colleen Cutler, Waterloo, Ontario.

There will be a special session in collaboration with the *International Association of Official Statistics* and a round table discussion on the role of statistics in transition countries. Among the participants are T. Toczyński, President of the Polish National Bureau of Statistics, Sten Johansson, Professor of Sociology and former Director General of Statistics Sweden, Willem de Vries, Deputy Director of Statistics Netherlands and P. Adlys, Acting Director, Lithuanian Statistics Bureau. The organizers are Petras Adlys and Rimantas Rudzkis.

In connection with the meeting there will be a *Workshop on Stochastic Analysis*, organized by David Elworthy.

Some ninety scientists from all over the world have already accepted invitations to speak at the ordinary sessions. The sessions have been listed in the last issue of Bernoulli News.

Persons wishing to present a contributed paper are requested to present the abstract of the paper (no more than two pages) written in English. Please indicate the title of the session which is closest. Abstracts of contributed papers, papers and registration forms are to be sent to the Organizing Committee by January 15, 1998. Registration forms will appear in the first bulletin, to be distributed soon.

The registration fee is 150 USD. It is intended to reduce the fee for participants from developing countries.

Accomodation will be available in the hotels *Lietuva and Zaliasis Tiltas*, which are located in the center of the city, at hotel *Draugyste* and at budget prices at the Hotel and Student Dormitory of the Technical University on the University campus (the conference site).

The Organizing Committee intends to arrange a special conference bus from the hotels to the conference site.

For more information please contact:

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History of Probability and Statistics at the International Congresses of Mathematicians

Ulrich Krengel, University of Göttingen, Germany

The first International Congress of Mathematicians (ICM) was organized in 1897 in Zurich. Therefore, the 1994 ICM, again in Zurich, was almost a centennial. For this reason, an International Congress on the History of Mathematics was organizied by Prof. Dr. E. Neuenschwander (Univ. Zurich) at the same time and location as the 94 ICM. Five mathematicians were asked to speak on the history of the ICMs in five different mathematical fields. The present article is a slightly modified version of the lecture on probability and statistics given by the author.

We ask which important developments in probability and statistics have been represented at the ICMs, and in which way ICMs can serve other aims than special conferences. We hope to offer constructive criticism. Some of the findings concerning certain classical results and notions in probability do not seem to be common knowledge yet.

We start with a very general question:

what are the legitimate aims of the International Congresses? Have these aims been pursued and achieved in the fields of probability and statistics?

There are three aims of scientific congresses which apply to all congresses:

(1) *gathering of information and stimulation in one's own field,*

(2) *presentation of one's own results,*

(3) *establishment of personal contacts and personal discussions.*

These first three aims can certainly be achieved best by specialised conferences, for example, by conferences devoted only to probability and/or statistics or even to subfields of these. Aims more specific to the ICMs may be the following.

(4) *Improvement of the knowledge of mathematicians about developments in mathematics outside their own field.*
This is a basic cultural aim. More practically, it can contribute to a fair evaluation of the importance of other fields, and it can also help to give new directions to one's own work.

(5) *Improvement of public relations.*
The public and also the funding institutions should be convinced of the relevance of mathematical research including the importance of purely theoretical work.

(6) *Hero–worship, and recognition of merit.*
This aim has been rather dominant in practice. (In my view, hero–worship may be a legitimate aim to some extent as a way of achieving (4) and (5)).

The role played by probability and statistics at the ICMs has changed a lot in the course of the last century. If we want to form a judgement on whether the above aims were met we must differentiate between different time periods. After looking at the Proceedings of all the ICMs, I felt that they divided naturally into three markedly distinct periods, namely:

1897–1924 (Zürich – Toronto)
the formative years
1928–1936 (Bologna – Oslo)
the foundational years
1950–now (Cambridge/Mass. – Zürich)
the flourishing years.

(Because of the world war, there were no ICMs between 1936 and 1950.)

1897–1924 The formative years

During the first quarter of this century, probability and statistics had a very bad reputation among general scientists as well as mathematicians due to the lack of rigorous foundations. Usually, there were sections on probability, statistics and actuarial mathematics at the ICMs, largely devoted to special problems in actuarial mathematics that left no traces in the field. At **Zürich** (1897) there was not a single talk in probability. (E. Borel, F. Hausdorff and E. Czuber were among the participants.)

In 1890, at **Paris**, the most remarkable event related to probability was the fact that *Hilbert* posed the problem of specifying axiomatic foundations of probability. This was the main part of the sixth of his famous 23 problems. It is not unlikely that the idea of treating probability axiomatically first occurred to Georg Bohlmann (1869 - 1928). He was a Privatdozent in Göttingen since 1894, and he taught actuarial mathematics. He wrote a long article [B] on life insurance in the Enzyklopädie der Mathematischen Wissenschaften, and this article started with an axiomatics of probability. This material has been first presented in Paris (1900) during an actuarial congress. Hilbert refered to an announcement of this work when he posed the problem to axiomatize probability. It is well known that Hilbert consulted with various colleagues prior to his Paris talk. Bohlmann himself asserted that Hilbert's axiomatic treatment of geometry inspired him, but not that Hilbert posed the problem. Bohlmann introduced the probability $p(E)$ of an event E as a nonnegative number such that the certain event has probability 1 and such that (finite) additivity holds. He made a point that he tried an approach which was basically different from that of Laplace. His axioms turned out to be unsatisfactory since he did not have a rigorous definition of an event and he was aware of this deficiency. Yet, the spirit of his approach certainly resembles the present axiomatics of Kolmogorov more than the ideas which had been used in the previous century. (It seems that Kolmogorov was never told about Bohlmann though he repeatedly visited Göttingen prior to publishing his "Grundbegriffe" in 1933. We remark that Ugo Broggi (1880 - 1965), in a thesis written under the direction of Hilbert, defined events as subsets of a fixed set in 1907. Broggi also introduced σ-additivity, but gave a wrong proof of its (supposed) equivalence to finite additivity.

It seems that the colleagues of Bohlmann did not appreciate his ideas and ridiculed his attempts to make probability rigorous. According to Kamke [Ka] the following sentence circulated in Göttingen at the beginning of this century: "A probability is a number between 0 and 1 about which nothing else is known." In 1902, Bohlmann accepted a position in an insurance company.

At **Heidelberg** (1904), there was a section on applied mathematics, but no talk on probability or statistics was given.

At **Rome** (1908) the section on probability, statistics and actuarial mathematics was resumed. There were eleven talks on special subjects in actuarial mathematics. *E. Borel* gave a talk "Sur les applications du calcul des probabilités aux sciences biologiques". *G. Bohlmann* talked about applications of probability to actuarial mathematics. In his article for the Rome ICM, Bohlmann gave the present formal definition of independence of n events: "The events $A_1, ...A_n$ are *called* independent if for any subset of them the probability of their joint occurrence is the product of their probabilities." Of course, the "product formula" had been used for a long time, but it had been considered a provable result, not a definition (Multiplication theorem of probabilities). Moreover, it was not obvious that one had to require the validity of the product formula for all subfamilies of the considered family of events. Bohlmann showed by example that pairwise independence did not imply independence. Kolmogorov, in his "Grundbegriffe", gave credit to Khinchine for the formal definition of independence, quoting an article of 1927. In most probability books, the example showing that pairwise independence does not imply independence is still attributed to Khinchine. Apparently, nobody looked at the proceedings of the ICM of 1908.

Among the other speakers at Rome, I mention the statistician *C. Gini* who spoke on "rare events".

As *A.M. Ljapounov* was a member of the international committee organizing the ICM in **Cambridge/UK** (1912), it is surprising that the remarkable progress of the Russian probabilists still found no way into the program. In *F.Y. Edgeworth's* opening address for the section on probability and statistics there was a remark "Calculus of probability has been described as opprobium (shame) of mathematics". (This was a quotation of a statement of John Stuart Mill (1846), who had only referred to the application of probability to social science topics like the testimony of witnesses (see [S1], p. 195.)). Edgeworth expressed his hope that the talks, and the recent creation of the position of a lecturer of statistics in Cambridge might help to overcome this disapproval of probability. Again, many talks on actuarial mathematics were presented. *F.Y. Edgeworth* gave a talk "A method of representing statistics by analytic geometry".

During the first world war, there was no ICM.

At **Strasbourg** (1920), the only talk related to probability or statistics was given by *A. Guldberg.* He spoke on "Une application des polynômes d'Hermite à un problème de statistique."

J.C. Fields, the president of the **Toronto** ICM (1924) (who initiated the creation of the medals, later named after him against his his own wish) wanted to accentuate, more than at previous congresses, the role of applied mathematics. He hoped that the Congress would not be without influence on the layman "to whom science must ultimately look for its material support". Therefore, he organized sections on electrical, mechanical, civil, and mining engineering as well as on aeronautics, naval architecture, ballistics and radiotelegraphy. It is therefore not surprising that there was also a large section on statistics and actuarial science. The talk by *R.A. Fisher* "On a distribution yielding the error functions of several well–known statistics" may be the first ICM talk giving results of historical significance in statistics. He discussed many topics appearing a year later in his well–known monograph "Statistical Methods for Research Workers": the F–distribution, analysis of variance and multiple correlations.

One paper in the Toronto Proceedings shows how the lack of foundations of probability made it impossible to identify ill–defined problems.
E.B. Wilson discussed a problem from Keyne's "Treatise on Probability": If k successes have been observed in n trials, what is the probability that k' successes shall be observed in the next n' trials? (The problem is, in fact, much older, see [S2].)

We conclude: During the period 1897–1924, probability and statistics were still in rather bad shape. The main contributions by Ljapounov, Markov, Khintchine, Borel, Pólya, Lindeberg, Wiener, Lévy, K. Pearson, and others were not presented at the ICMs.

1928–1936: The foundational years
The 1928 ICM at **Bologna** was the first at which a plenary lecture dealt with probability. *E. Borel* announced a talk "Le Calcul des Probabilités et les Sciences Exactes". (As Borel was unable to appear, his manuscript was read by E. Cartan.) His point was that probability and statistics were no less important in the exact sciences than in biology and the social sciences. He named important applications: the theory of errors, actuarial mathematics, the kinetic theory of gas, astronomy, and radioactive phenomena. Certainly, the fact that an eminent mathematician like Borel devoted his plenary lecture to emphazise the importance of probability and statistics must have made quite an impression. On the other hand, the talk did not report any mathematical results. This may have appeared like a confirmation of the existing prejudices.

De Finetti presented the first systematic study of exchangeability at Bologna. *Hadamard* spoke on card shuffling and statistical mechanics (in addition to his invited talk on functional analysis). [For lack of space, further contributed talks must remain unmentioned.]

Among the shorter talks, that of *J. Neyman*, who explained the idea of likelihood ratio tests developed jointly with E. Pearson, and a talk by *R.A. Fisher* on maximum likelihood and χ^2–tests seems particularly important.

One event at the Bologna ICM has recently been discussed in Historia Mathematica by E. Seneta [Se]: There was a heated discussion between *F.P. Cantelli* and his followers on one side and *E.E. Slutsky* on the other. Slutsky was attacked since, in one of his articles, he had written that the first mathematician who *investigated* the strong law of large numbers was Emile Borel, and that it was then studied by Cantelli, Khinchine and others. The italians insisted that the first strong law was proved in 1917 by Cantelli [Ca].

Perhaps, one reason for the controversy was the fact that, in 1909, Borel [Bo] had studied dyadic expansions and not coin tossing, and the equivalence of the two problems was not clear at that time. More significantly, Borels proof contained two nontrivial gaps. He had proved the Borel-Cantelli lemma for independent events, but applied it to dependent events. His proof used the normal approximation of the binomial distribution as an exact formula, ignoring the need to estimate the difference between the binomial distribution and the normal distribution. Cantelli gave a correct argument which applied to general random variables with finite fourth moments. (As he did not assume σ–additivity, he could not assert almost sure convergence, but his result implied almost sure convergence in the presence of σ–additivity.)

Seneta regrets that Barone and Novikov [BN], in an important historical article on the history of the axiomatic foundation of probability, attribute almost all of the credit for the strong law to Borel and Hausdorff. However, he does not inform his readers what Hausdorff did. The fact is that Hausdorff, in 1914, in his famous book on "Mengenlehre" [H], gave the first rigorous proof of the theorem stated by Borel. Like Cantelli's proof, given three years later, his argument was based on an estimate of the fourth moments and a Chebyshev-Markov type inequality, but it was restricted to dyadic expansions. Implicity, it contained a proof of the dependent half of the Borel-Cantelli lemma in the special case he needed. Though he worked in the dyadic expansion setting, he called the theorem "a plausible extension of the law of large numbers to the infinite case". It therefore seems clear that he was aware of the fact that the theorem had a probabilistic interpretation. Hausdorff, apparently a generous gentleman, did not mention the gaps in the proof of Borel and the fact that he gave an entirely different argument. He used the term "theorem of Borel". In my view, the term "theorem of Borel-Hausdorff" would be better justified.

Cantelli quoted neither Borel nor Hausdorff. Perhaps, Slutsky and Cantelli would have avoided their dispute if they had been aware of the work of Hausdorff.

At **Zurich** (1932), *Serge Bernstein* gave a plenary talk "Sur les liaisons entre quantités aléatoires". He reported about numerous new results both by himself and by other Russian probabilists. For example, he presented a central limit theorem under asymptotic independence conditions, and he discussed exchangeable sequences of events, Markov chains and Brownian motion. He also spoke on Kolmogorov's results concerning continuous time Markov processes and the Fokker–Planck equation.

Richard von Mises gave an invited talk in the section on probability, statistics and actuarial mathematics. He first discussed his axiomatics, admitting that he might not yet have found the final formulation. Then he, too, dealt with several of the recent results of Khintchine, Kolmogorov and others such as the law of the iterated logarithm and the strong law of large numbers.

In **Oslo** (1936) *Maurice Fréchet* devoted a portion of his plenary lecture "Mélanges mathématiques" to topics in probability. In addition to his own work, he discussed progress by Hadamard, Hostinský, Doeblin and Kolmogorov. He also stressed the fact that by now probability was equally exact as other branches of mathematics, and that many mathematical subjects (integral equations, group theory, etc.) were used as tools in probability. Another important talk in Oslo was the one given by *W. Feller* on stochastic processes. He discussed his well–known existence and uniqueness theorems for Markov processes with jumps. *Harald Cramér* announced his asymptotic expansions and estimates related to the central limit theorem.

Perhaps one can characterize the period between the ICMs at Bologna and Oslo as follows: The growing importance and increased rigor of probability and statistics became obvious. On the other hand, probability was not yet fully accepted. Some of the principal contributors were ignored. For example, the important work of Paul Lévy on the central limit problem and on characteristic functions found no mention. We know from the autobiography of Paul Lévy, that he had a very different point of view from that of Borel. He felt that probability had to be based on rigorous analysis, while Borel felt that it should be studied from the point of view of applications. Thus, Borel may actually have delayed the acknowledgement of probability as a proper mathematical discipline. The work of Kolmogorov and Khintchine was only reported by others. Other names that I missed are Steinhaus, Pollaczek, Marcinkiewicz, and Zygmund.

1950–now The flourishing years

Due to the second world war, it took fourteen years before the next ICM took place in **Cambridge, Massachusetts** in 1950. During this time there had been tremendous progress in probability, statistics and related fields, and the organizers managed to present an impressive overview of this development. In spite of the fact that – because of the cold war – the Russian mathematicians were not allowed to go to America , the list of speakers reads like a "Who's Who in Probability and Statistics".

There were three pertinent plenary lectures, including the first one devoted to mathematical statistics. *Abraham Wald* spoke on: "Basic ideas of a general theory of statistical decision rules". Statistics was presented as the theory which had to compare decision functions and to select those for which the risk function was minimal in a suitable sense. This theory certainly appealed to mathematicians in that clear optimization questions were asked. It unified testing and estimation problems. Wald discussed also the sequential probability ratio test, which had been kept secret during the war because of its practical value in quality control. Since Wald died in December 1950, this must have been one of his last talks.

S. Kakutani gave a plenary lecture on "Ergodic Theory". His talk was a survey starting with the ergodic theorems of von Neumann and Birkhoff, and continuing with the work of Yosida–Kakutani and with Wiener's multiparameter ergodic theorems. Moreover, he discussed the analogies with the martingale convergence theorems of Doob, Lévy, and Ville. In my view such a broad historical view must have been very attractive not only to specialists.

N. Wiener spoke on prediction theory, presenting his results obtained during the war independently of Kolmogorov. (He had given much attention to the practical problem of finding the prediction explicitly in a form useful for electrical engineering.)

In additon to these plenary lectures, there was a section on probability, statistics and related fields chaired by *Doob*, and a Conference on Applied Mathematics chaired by *J. von Neumann*, also offering some probability. In the Doob section the principal invited talks were:

R.C. Bose:	Mathematics of factorial designs
P. Lévy:	Processus á la fois stationnaires et Markoviens pour les systèmes ayant une infinité dénombrable d'états possibles
S.N. Roy:	Some aspects of statistical inference.

But also the list of speakers for the contributed papers was very impressive. It included *Wolfowitz, Robbins, Shapley, Tukey, Chung, Yosida* and *ergström.*

In the Conference chaired by von Neumann, there were three talks related to probability:

C.E. Shannon:	Some topics in information theory
S.M. Ulam:	Random processes and transformations
W. Feller:	Some recent trends in the mathematical theory of diffusion.

Certainly the last of these was a highlight. Feller explained the ties between classical boundary problems for the heat equation and diffusion processes. He also spoke on Ito's theory of stochastic integration. This was possibly the first time that these topics were presented to a broad mathematical public.

The ICM was preceded by the important *Second Berkeley Symposium on Probability and Statistics.* Within one summer the explosion of progress in probability and statistics was demonstrated at two major international conferences. After these events no serious mathematician could argue anymore that probability and statistics were not important branches of mathematics.

By 1954, at the time of the ICM in **Amsterdam**, *Kolmogorov* was a legend in probability, who had, for about thirty years, produced numerous works of historical significance. He was now for the first time invited to deliver a plenary address at an ICM: But at this time he was working primarily on dynamical systems, initiating his share of the Kolmogorov–Arnold–Moser–theory. He decided to speak on dynamical systems. Also, later, he never gave an ICM talk on probability.

But by now probability and, to a lesser degree, statistics had become a significant part of the ICM–programs. For this reason, I can give only kind of a list of the subsequent main contributors with some limited comments.

Jerzy Neyman gave a plenary address: "Current problems of mathematical statistics". His aim was to give a general characterization of mathematical statistics understood as an independent mathematical discipline, to treat its contacts with the experimental sciences, and to indicate several directions of research. He discussed the Neyman–Pearson theory of testing hypothesis, models of clustering, identifyability of models, efficiency and superefficiency, sufficient statistics and uniformly most powerful tests.

J.L. Doob gave an address on "Interrelations between Brownian motion and potential theory" presenting the connections between diffusion processes and parabolic differential equations, martingales and submartingales, the material of his key paper in the 1954 AMS Transactions.

David Blackwell spoke on "Controlled random walks". He explained his analog of the minimax theorem for vector payoffs which now plays an important role in the theory of repeated games.

There was, again, a *Symposium on Stochastic Processes* linked to the ICM. Doob spoke on "Present state and future prospects of stochastic process theory", indicating important directions of future research. R.M. Fortet spoke about joint work with E. Mourier on probability in Banach spaces. Paul Lévy presented the "Processus semi-Markoviens" introduced independently by W.L. Smith. Other well–known speakers included A. Rényi (on order statistics and on axiomatics), D.G. Kendall, G.E.H. Reuter, E. Hille, and D.V. Lindley.

In **Edinburgh** (1958), *Willy Feller* gave a plenary talk "Some new connections between probability and classical analysis". Starting with an elementary classical setting he gave a scholarly motivation of Martin boundaries.

B.V. Gnedenko gave a survey on "Limit theorems in probability theory" including topics like fluctuation theory, local limit theorems, sums of functions of Markov chains, Berry–Esseen type estimates. In view of the broad scope of the talk few results were made explicit.

K.L. Chung spoke on "Continuous parameter Markov chains", indicating many open problems. *A. Rényi* gave a talk on probabilistic methods in number theory, treating the normal approximation for the number of prime factors, the large sieve of Linnik, and continued fractions. On the statistics side, *Leonard J. Savage* spoke on "Recent tendencies in the foundations of statistics" stressing the economic point of view and the subjective point of view.

In **Stockholm** (1962) *E.B. Dynkin* presented a plenary lecture "Markov processes and problems in analysis". He treated the characteristic operator of a Markov process, the notion of additive functional, and various transformations of Markov processes. Moreover he discussed problems related to the class of all nonnegative harmonic functions.

G.A. Hunt lectured on "Transformations of Markov processes". He described a direct construction of the entrance boundary for a Markov process on a locally compact space. Ya.G. Sinai spoke on "Probabilistic ideas in ergodic theory". He treated entropy of measure preserving transformations and K–systems. (The principal result, the weak isomorphism theorem for Bernoulli shifts remained unclear for many years.)

K. Itô spoke on Brownian motion on Riemannian manifolds. There were shorter communications by *Furstenberg, Hoeffding* and *Linnik.*

At **Moscow** (1966), there were two plenary lectures of interest to us: *Richard Bellman* talked on

"Dynamic programming and modern control theory". He explained the basic idea of dynamic programming, and gave a brief outlook at the stochastic case hinting at sequential analysis and prediction theory. Most of his talk though, was not related to probability. *Charles M. Stein* lectured on "Some recent developments in mathematical statistics". His talk treated admissible estimators (including his paradox), invariance of decision precedures, similar tests, and large sample theory.

The main talks in the probability section were given by *V. Strassen* ("The theorem of the iterated logarithm") and by *A.A. Borovkov* ("On conditions of convergence of diffusion processes and asymptotic methods of queuing theory"). *Strassen* started with a historical survey on the work of Khintchine, Kolmogorov–Petrovsky–Erdös and reported his invariance principle and his precise studies about the instant of last excedence of Brownian motion of suitable functions $\varphi(t)$, which he had recently presented at the 5^{th} Berkeley Symposium.

At **Nice** (1970), the principal talks in the section on statistics were those by *J. Kiefer, Yu.V. Linnik, L.N. Bolshev* and *J. Wolfowitz. J. Kiefer* gave a survey on 20 years of work on optimum experimental designs starting with Mood, Elfving, Chernoff and leading to his own work. *Linnik* spoke on sequential estimation. The talk by *Bolshev* appears to me as a particularly good example of exposition to nonspecialists. He spoke on the empirical Bayes approach, starting with simple examples, motivating the background and illustrating the traps. *Wolfowitz* presented the theory of maximum probability estimators, which he had developed with Lionel Weiss.

The section on probability included talks by *D.L. Burkholder, R.V. Chacon, A. Dvoretzky, D. Ornstein, V.V. Sazonov* and *M. Sion. Burkholder* presented his inequalities for operators on martingales, a subject of great importance in martingale theory. *Chacon* spoke on "Representation of measure transformation", a topic in ergodic theory. *Dvoretzky* discussed his work on the central limit theorem for dependent random variables. The results of *Ornstein* on "Entropy is enough to classify Bernoulli shifts but not K–automorphisms" remain a celebrated breakthrough in ergodic theory, now included in many books. *Sazonov* spoke on the speed of convergence in the central limit theorem in the multidimensional case.

In addition to the probability section, there was a section on potential theory and Markov processes. Many of these talks treated the boundary theory of Markov processes. The speakers included *K.L. Chung, E.B. Dynkin, B. Fuglede, R.K. Getoor, H. Kesten, G. Mokobodski, A.N. Shiryaev* and *T. Watanabe.* Surely, the talk by *Kesten* on "Hitting of sets by processes with stationary independent increments" was a highlight of the section in that it treated hard problems which were very concrete and could be understood by nonspecialists.

Heinz Bauer's plenary talk at **Vancouver** (1974) on potential theory included some probabilistic aspects, but they were not the main theme. Compared to the preceeding ICM, the section on probability and statistics was reduced. *R.V. Ambartzumian* spoke on "The solution of the Buffon–Sylvester problem and stereology". It seems that this was the first ICM–talk on stochastic geometry, the study of random sets for which the law is invariant under certain groups of transformations. Stereology deals with inference on the size and shapes of bodies in 3–dimensional space, based on random plane sections. *R.M. Dudley* talked on Gaussian processes, giving conditions for sample continuity in terms of metric entropy. He also gave conditions for central limit theorems in B–spaces. *J. Neveu* presented results of Brunel, Revuz, Métivier and, last but not least, himself on potential theory for recurrent Markov processes. *C.R. Rao* discussed the characterization of distributions through properties of sample statistics. *Frank Spitzer* gave a survey of the work of Dobrushin, Harris, Holley, Liggett, himself and others on Markov random fields and infinite Gibbs states. This area of research has flourished since. *V. Statulevicius* spoke on limit theorems for dependent random variables, *B. Walsh* on harmonic spaces, and *J. Walsh* on stochastic integrals in the plane. Finally, *Peter J. Huber* gave a very well motivated introduction to basic questions in robust statistics, starting with qualitative requirements as introduced by F. Hampel, and leading to the generalized Neyman-Pearson lemma for capacities due to Huber and Strassen.

At **Helsinki** (1978), again, the organizers had to cope with political difficulties. Not all invited speakers from the Soviet union were allowed to attend the congress, see [Al]. For example, *R. Dobrushin* did not give his announced plenary lecture on "Classical statistical mechanics as a branch of probability". On the other hand, *A.N. Shiryaev* was able to present his plenary address on "Absolute continuity and singularity of probability measures in functional spaces".

In the section on probability and statistics, *A.A. Borovkov, A.D. Wentzell* and *S.R. Varadhan* treated various problems on large deviations. Notably, Varadhan's results have had a lot of impact. *C. Dellacherie* gave an overview of the theory of stochastic integrals. *M. Fukushima* spoke on Dirichlet spaces and additive functionals of finite energy. *P. Révész* presented his joint work with Erdös on limit theorems for the length of the longest head run in coin tossing, and related problems. In the economics section, the talk of *Robert J. Aumann* on recent developments in the theory of Shapley value certainly was of interest to probabilists. It seems that there was no invited talk on statistics.

Two of the plenary lectures at **Warsaw** (1983) were related to probability. *Wendell H. Fleming* talked on "Optimal control of Markov processes". He explained what optimal control theory is about and described the key methods. He also sketched some cases in which solutions exist, without, however, emphasizing particular results. *Aleksander Pelczyński* spoke on "Structural theory of Banach spaces and its interplay with analysis and probability". He mainly explained what the modern theory of Banach spaces is about, just barely touching probability.

David Brillinger gave a talk on statistical inference for random processes, discussing the whole inferential process; from the data collection and the-modelling, through the inferences themselves. This time, there was even a second talk on statistics. *D.M. Chibisov* discussed "Asymptotic expansions and deficiencies of tests".

H. Kesten gave an introduction to bond percolation on the lattice and the determination of the critical percolation probability. *Paul Malliavin* spoke on his calculus, *Petr Mandl* on self–optimizing control of Markov processes and Markov potential theory, *Daniel W. Stroock* on stochastic analysis and certain partial differential operators, *S. Watanabe* on "Excursion point processes and diffusions", and *J. Zabczyk* on stopping problems for continuous time Markov processes in stochastic control theory. On the whole, stochastic analysis played a rather dominant role.

In 1986, the ICM was organized in **Berkeley**. As a plenary speaker, *A.V. Skorohod* spoke on "Random processes in infinite dimensional spaces". There were seven invited talks in probability/statistics sections. *T. Arak* described a class of Markov fields with finite range. *M.H.A. Davis* gave a careful survey on nonlinear filtering theory. *A.S. Holevo* spoke on quantum probability, and *H. Kunita* on stochastic flows and stochastic partial differential equations. The talk of *T.M. Liggett* was on "Spatial stochastic growth models – survival and critical behaviour". In this area, the questions have a lot of intuitive appeal, so that also non–probabilists must have been able to get a good idea of the field. *G.C. Papanicolaou* dealt with "Wave propagation and heat conduction in random media". The only statistics talk was presented by *C.J. Stone*. He spoke on a nonparametric setting with unknown smooth densities, giving an asymptotic theory on reducing the nonparametric setting to a parametric one with finite–dimensional approximations.

The spectrum of the probability/statistics talks at the **Kyoto**–ICM in 1990 was quite broad: *Persi Diaconis* spoke on "Applications of group representations to statistical problems". He discussed the handling of data related to a finite group acting on a finite set, employing a splitting of the space of functions on that set into components from invariant subspaces. Examples arise for instance in time series and the modern analysis of variance. The relevance for applications became quite clear from his exposition.

Martin T. Barlow introduced ideas on ''Random walks and diffusions on fractals ". The background is heat conduction on fractal sets, and, more generally, second order partial differential equations on fractal spaces. One constructs random walks converging to a diffusion process on the set.

Richard Durrett gave a survey on stochastic models of growth and competition like the competition of plant species or the spread of an epidemic. Examples are contact processes, multiple contact processes and more hierachical models like bushes and trees in the plane.

Hillel Furstenberg gave an overview of ergodic theoretic methods in Ramsey theory. He treated for example joint work with Katznelson on long arithmetic progressions (Szemerédi type theorems) and coloring theorems.

Lucien Le Cam contributed an article on "Recent results in the asymptotic theory of statistical estimation", treating for example the Hájek–Le Cam asymptotic minimax theorem.

Other talks were given by *S. Kotani* on random Schrödinger operators, *S. Kusuoka* on "De Rham cohomology of Wiener–Riemannian manifolds, *S.A. Molchanov* on "Localization and intermittency". *Marc Yor* discussed the laws of some Brownian functionals such as Paul Lévy's stochastic area of planar Brownian motion.

I think it is clear from these descriptions that probability and statistics regularly contributed in a rather profound and rich way to the programs of the ICMs since 1950. The mathematical level was very high. However, this does not necessarily imply that the aims proposed at the outset were well achieved.

First of all, it seems important to me to put more weight on applied work. One reason, of course, is that we need to explain to the public and to various institutions that there is some use for what we are doing. Another, possibly more important reason is that we must make it clear to ourselves that good motivation is an essential ingredient of good mathematics. If a mathematical result is hard, this does not automatically imply that the result is worthwile. I hope that I am not misunderstood: I am very much for fundamental research in pure mathematics. We must understand the structure of the leading notions. Routine applications of standard methods are not good applied mathematics. But surely a man like Claude Shannon, creating information theory, did more for mathematics than some of the Field's medalists. It seems to me that virtually all the Field's medalists view themselves as pure mathematicians. This exclusive admiration of pure mathematics is very questionable. Surely, it is not in the spirit of J.C. Fields, although he was a pure mathematician himself. The feeling of doing something which is ultimately useless is quite widespread among mathematicians. Therefore, we need examples of good applied work. We must re–examine our value system. Mathematical power is not the only ingredient needed to do good applied work. Creating the Neyman–Pearson work was not a question of mathematical power. However, it required careful analysis and sound judgement. The first difficulty was to ask the right question. In the end, their work led to an understanding of some of the principal mathematical notions affecting our daily life.

Another example: stochastic differential equations or martingales are now used in many applied fields, not just by mathematicians. Certain new notions and theories have more impact than the solution of some old hard problems. I gladly agree that the recent astounding progress with many classical problems deserves our highest admiration. But there should be an effort to recognize also other breakthroughs. Perhaps, more balance has been achieved recently in Zurich.

A second point I wish to make is that, even in the plenary talks, many speakers aim at impressing the audience rather than at explaining their problems, motivations and ideas. Reading the Proceedings of the ICMs, I sometimes had difficulties even with talks in probability. If I have a hard time, what will be the situation of mathematicians working in other fields? If I listen to a talk in another field, I am frequently lost quite rapidly. If the purpose of the ICMs is the communication between different branches of mathematics, the organizers must put great pressure on the speakers to make their talks simple to understand even for colleagues working in entirely different fields. They should also make it clear to the speakers that a survey including some historical background is preferable to an unpublished special new result, even if this result is a very good one. ICMs serve a different purpose from specialized meetings. In view of this purpose, I would prefer a speaker who explains the work of some other mathematicians well to a speaker who solved a big problem but is unable to communicate his ideas. I admit that it may be a great experience to see a famous mathematician on the platform even if his talk is difficult to follow. But it is never nice to listen to a bad talk. All too often an extremely high proportion of an audience is lost very early into a talk. The selection of the speakers need not primarily be a method of naming the heroes of our field. The interest of the audience deserves priority.

A rather obvious observation is that statistics has been grossly underrepresented at the ICMs. It may not be possible to attract large numbers of mathematical statisticians to the ICMs. They find more stimulation valuable to them in statistics conferences. But mathematicians should learn something about fields in which many of their students finally work. If we want to preserve some unity of mathematics, we should strive to prevent branches like statistics, operations research, actuarial mathematics from drifting completely away from other branches of mathematics. Otherwise, they may soon not be served by us any more. A good way to attract more specialized groups periodically has been to organize satellite conferences.

Finally, it may be of interest to ask which topics in probability and statistics have been neglected. This is a very subjective matter. It seems to me that the coverage in probability has been quite broad. Perhaps, some topics like boundary theory or stochastic analysis received more than their fair share at some of the meetings. Other topics like renewal theory, branching processes, queuing, point processes, information theory, random search, ruin theory, game theory, multiparameter processes, strong approximations of random walks, or probabilistic algorithms received little attention. (Sometimes, special advanced results from some of these areas may have been treated, but a survey might have been more appropriate for a general audience.)

In statistics, topics like sequential analysis, nonparametric statistics, empirical processes, survey sampling, multivariate analysis, bootstrap, stochastic approximation have been largely absent. Perhaps, we will see some of these topics next time.

I would have liked to compare the results presented at the ICMs with those presented in some history of probability and statistics of this century. Unfortunately, this history has to be written yet. There are, however, some articles treating parts of the history and personal recollections, which I have consulted. The interested reader is referred to [C], [Do], [G], [L1], [L2], [S1], and [Sh].

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